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Generalized magneto-thermoelasticity in a perfectly conducting medium

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Abstract

A model of the equations of generalized magneto-thermoelasticity in a perfectly conducting medium is given. The formulation is applied to generalizations, Lord–Shulman theory with one relaxation time and the Green–Lindsay theory with two relaxation times, as well as to the coupled theory.

Laplace transforms and Fourier transforms techniques are used to get the solution. The resulting formulation is used to solve a specific two-dimensional problem. The inverses of Fourier transforms are obtained analytically.

Laplace transforms are obtained using the complex inversion formula of the transform together with Fourier expansion techniques.

Numerical results for the temperature distribution, thermal stress and displacement components are represented graphically. A comparison was made with the results predicted by the three theories.

Keywords: Elasticity; Thermoelasticity and magneto-thermoelasticity

1. Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms. Second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves.

Boit (1956) introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory.

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Nomenclature

λ, μ	Lame's constants
ρ	density
C_E	specific heat at constant strain
t	time
T	absolute temperature
T_0	reference temperature
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
k	thermal conductivity
μ_0	magnetic permeability
ε_0	electric permeability
a_0^2	$=\frac{\mu_0 H_0^2}{\rho}$, Alfen velocity
c^2	$=\frac{1}{\mu_0\varepsilon_0}$, light speed
α	$=1+\frac{a_0^2}{c^2}$
β_0^2	$=\frac{\lambda+2\mu}{\rho}$, speed of propagation of isothermal elastic waves
c_0^2	$=\beta_0^2+a_0^2$
c_2	$=\sqrt{\frac{\mu}{\rho}}$, velocity of transverse waves
β^2	$=\frac{c_0^2}{c_2^2}$
c_1^2	$=\frac{c_0^2}{c^2}$
α_0	$=\alpha\beta^2$
η_0	$=\frac{\rho C_E}{k}$
τ_0	relaxation time
e	dilation
γ	$=(3\lambda+2\mu)\alpha_t$
ε	$=\frac{\gamma}{\rho C_E}$

However, both theories share the second shortcoming since the heat equation for the coupled theory is also parabolic.

Two generalizations to the coupled theory were introduced. The first is due to Lord and Shulman (1967) who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. Since the heat equation of this theory is of the wave-type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and the uncoupled theories.

The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Müller (1971) in a review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay (1972) obtained an explicit version of the constitutive equations. These equations were also obtained independently by Şuhubi (1973). This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Erbay and Şuhubi (1986) studied wave propagation in a cylinder. Ignaczak (1985) studied a strong discontinuity wave and obtained a decomposition theorem (Ignaczak, 1978). Ezzat (1995) has also obtained the fundamental solution for this theory.

The foundations of magnetoelasticity were presented by Knopoff (1955) and Chadwick (1957) and developed by Kaliski and Petykiewicz (1959).

An increasing attention is being devoted to the interaction between magnetic field and strain field in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics and related topics. In all papers quoted above it was assumed that the interactions between the two fields take place by means of the Lorentz forces appearing in the equations of motion and by means of a term entering Ohm's law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field.

Many authors have considered the propagation of electromagneto-thermoelastic waves in an electrically and thermally conducting solid. Paria (1962) discussed the propagation of plane magneto-thermoelastic waves in an isotropic unbounded medium under the influence of a uniform thermal field and with a magnetic field acting transversely to the direction of the propagation. Paria used the classical Fourier law of heat conduction, and neglected the electric displacement. Wilson (1963) extended Paria's results by introducing a component of the magnetic field parallel to the direction of the propagation. A comprehensive review of the earlier contributions to the subject can be found in Paria (1967). Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Namat-Nasser (1972) who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. They obtained the governing equations in the general case and the solution for some particular cases. Choudhuri (1984) extended these results to rotating media. Sherief and Ezzat (1996) solved a thermal shock half-space problem using asymptotic expansions. Lately, Ezzat (1997a,b) solved problems in a perfectly conducting medium, and Ezzat et al. (2000, 2001 and 2002) studied the propagation of plane waves in the same medium.

For this model, we solve a specific two-dimensional problem when the bounding surface of the half-space is taken to be rigid in x direction and no displacement in y direction. A thermal shock acts on a band of width $2a$ centered around the y-axis on the surface of the half space and is zero everywhere else. A magnetic field with constant intensity acts normal to the bounding plane.

2. Formulation of the problem

We shall consider a thermoelastic medium of perfect conductivity permeated by an initial magnetic field $H \equiv (0, 0, H_0)$. This produces an induced magnetic field $h = (0, 0, h)$ and induced electric field $E = (E_1, E_2, 0)$, which satisfy the linear equations of electromagnetism and are valid for slowly moving media of perfect electrically conductivity ($\sigma_0 \to \infty$) (see e.g. Ezzat, 1997a,b):

$$\operatorname{curl} h = J + \varepsilon_0 \dot{E}, \tag{1}$$

$$\operatorname{curl} E = -\mu_0 \dot{h}, \tag{2}$$

$$E = -\mu_0 (\dot{U} \wedge H), \tag{3}$$

$$\operatorname{div} h = 0. \tag{4}$$

These equations are supplemented by the displacement equations of the theory of elasticity, taking into account the Lorentz force

$$\sigma_{ij,j} + \mu_0 (J \wedge H) = \rho \ddot{U}_i, \tag{5}$$

and the heat conduction equation (see e.g. Ezzat and El-Karamany, 2002).

$$kT_{,ii} = \rho C_E(\dot{T} + \tau_0 \ddot{T}) + \gamma T_0(\dot{e}_{kk} + m\tau_0 \ddot{e}_{kk}) - (Q + m\tau_0 \dot{Q}), \tag{6}$$

where m is constant.

The constitutive equation

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma \widehat{T}\,\delta_{ij}, \tag{7}$$

where

$$\widehat{T} = T - T_0 + v\dot{T}, \tag{8}$$

and strain–displacement relations

$$\varepsilon_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}). \tag{9}$$

Together with the previous equations, constitute a complete system of generalized magneto-thermoelasticity for a medium with a perfect electric conductivity. Furthermore, it should be noted that the corresponding expressions for generalized magneto-thermoelasticity with one relaxation time deduced by setting $m = 1$ and $v = 0$, while for generalized magneto-thermoelasticity with two relaxation times deduced by setting $m = 0$, as well as for coupled theory by setting $v = \tau_0 = 0$.

In these equations, a dot denotes differentiation with respect to time, while a comma denotes material derivatives. The summation notation is used. We shall consider only the simplest case of the two-dimensional problem. We assume that all causes producing the wave propagation is independent of the variable z and that waves are propagated only in the xy-plane. Thus, all quantities appearing in Eqs. (1)–(9) are independent of the variable z. Then the displacement vector U has components $[u(x,y,t), v(x,y,t), 0]$.

Assume now that the initial conditions are homogeneous, then relation (1)–(3) yield (see e.g. Ezzat and Othman, 2000).

$$J = \operatorname{curl} h - \varepsilon_0 \dot{E}, \tag{10}$$

$$E = \mu_0 H_0(-\dot{v}, \dot{u}, 0), \tag{11}$$

$$h = -H_0(0, 0, e). \tag{12}$$

Expressing the components of the vector J in terms of displacement, by eliminating from Eq. (1) the quantities h and E and introducing them into displacement Eq. (5), Maxwell's equations become

$$J = \operatorname{curl} h - c_1^2 \dot{E}, \tag{13}$$

$$E = (-\dot{v}, \dot{u}, 0), \tag{14}$$

$$h = -(0, 0, e), \tag{15}$$

where $e = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$, the equations of motion have the form

$$\beta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + (\beta^2 - 1)\frac{\partial^2 v}{\partial x \partial y} - \beta^2\left(\frac{\partial \theta}{\partial x} + \upsilon \frac{\partial^2 \theta}{\partial x \partial t}\right) = \alpha_0 \ddot{u}, \tag{16}$$

$$\beta^2 \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + (\beta^2 - 1)\frac{\partial^2 u}{\partial x \partial y} - \beta^2\left(\frac{\partial \theta}{\partial x} + \upsilon \frac{\partial^2 \theta}{\partial y \partial t}\right) = \alpha_0 \ddot{v}, \tag{17}$$

in addition, the components of the stress are

$$\sigma_{xx} = \beta_0^2 \frac{\partial u}{\partial x} - (\beta_0^2 - 2)\frac{\partial v}{\partial y} - \beta^2\left(\theta + \upsilon \frac{\partial \theta}{\partial t}\right), \tag{18a}$$

$$\sigma_{yy} = (\beta_0^2 - 2)\frac{\partial u}{\partial x} - \beta_0^2\frac{\partial v}{\partial y} - \beta^2\left(\theta + \upsilon\frac{\partial \theta}{\partial t}\right), \tag{18b}$$

$$\sigma_{zz} = (\beta_0^2 - 2)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \beta^2\left(\theta + \upsilon\frac{\partial \theta}{\partial t}\right), \tag{18c}$$

$$\sigma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{18d}$$

$$\sigma_{xz} = \sigma_{yz} = 0. \tag{18e}$$

The heat equation

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \dot{\theta} + \tau_0\ddot{\theta} + \varepsilon(\dot{e} + m\tau_0\ddot{e}) - (Q + m\tau_0\dot{Q}). \tag{19}$$

These equations will be supplemented with appropriate boundary conditions relevant to the particular application under consideration as will be seen.

In the preceding equations, the following non-dimensional variables are used:

$$x = c_0\eta_0 x', \quad y = c_0\eta_0 y', \quad u = c_0\eta_0 u', \quad v = c_0\eta_0 v', \quad t = c_0^2\eta_0 t', \quad \tau = c_0^2\eta_0\tau',$$

$$J = \frac{J'}{\eta_0 H_0 c_0}, \quad \theta = \frac{\gamma(T' - T_0)}{\rho c_0^2}, \quad Q = \frac{\rho Q'}{kT_0 c_0^2\eta_0^2}, \quad h = \frac{h'}{H_0}, \quad E = \frac{E'}{\mu_0 H_0 c_0}, \quad \sigma_{ij} = \frac{\sigma'_{ij}}{\mu},$$

where the dashed quantities denote dimensional variables and it was canceled in the equations for convenient.

3. Formulation in the Laplace transform domain

We will apply Laplace transform defined as

$$\bar{f}(s) = \int_0^\infty f(t)\mathrm{e}^{-st}\,\mathrm{d}t,$$

hence, the above equations will take the forms

$$\beta^2\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + (\beta^2 - 1)\frac{\partial^2 \bar{v}}{\partial x\partial y} - \beta^2(1 + \upsilon s)\frac{\partial \bar{\theta}}{\partial x} = \alpha_0 s^2\bar{u}, \tag{20}$$

$$\beta^2\frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial x^2} + (\beta^2 - 1)\frac{\partial^2 \bar{u}}{\partial x\partial y} - \beta^2(1 + \upsilon s)\frac{\partial \bar{\theta}}{\partial y} = \alpha_0 s^2\bar{v}, \tag{21}$$

$$\frac{\partial^2 \bar{\theta}}{\partial x^2} + \frac{\partial^2 \bar{\theta}}{\partial y^2} = (s + s^2\tau_0)\bar{\theta} + \varepsilon(s + s^2 m\tau_0)\bar{e} - (1 + m\tau_0 s)\overline{Q}, \tag{22}$$

$$\bar{\sigma}_{xx} = \beta_0^2\frac{\partial \bar{u}}{\partial x} - (\beta_0^2 - 2)\frac{\partial \bar{v}}{\partial y} - \beta^2(1 + \upsilon s)\bar{\theta}, \tag{23}$$

$$\bar{\sigma}_{yy} = (\beta_0^2 - 2)\frac{\partial \bar{u}}{\partial x} - \beta_0^2\frac{\partial \bar{v}}{\partial y} - \beta^2(1 + \upsilon s)\bar{\theta}, \tag{24}$$

$$\bar{\sigma}_{zz} = (\beta_0^2 - 2)\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}\right) - \beta^2(1 + \upsilon s)\bar{\theta}, \tag{25}$$

$$\bar{\sigma}_{xy} = \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right). \tag{26}$$

4. Formulation in the Fourier transforms domain

We will apply Fourier transform defined as

$$f^*(q) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}qy} f(y)\,\mathrm{d}y,$$

where

$$f(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}qy} f^*(q)\,\mathrm{d}q.$$

Then, the last equations will be in the following forms:

$$\beta^2 \frac{\partial^2 \bar{u}^*}{\partial x^2} - (q^2 + \alpha_0 s^2)\bar{u}^* - \mathrm{i}q(\beta^2 - 1)\frac{\partial \bar{v}^*}{\partial x} - \beta^2(1 + \upsilon s)\frac{\partial \bar{\theta}^*}{\partial x} = 0, \tag{27}$$

$$\frac{\partial^2 \bar{v}^*}{\partial x^2} - (q^2\beta^2 + \alpha_0 s^2)\bar{v}^* - \mathrm{i}q(\beta^2 - 1)\frac{\partial \bar{u}^*}{\partial x} + \mathrm{i}q\beta^2(1 + \upsilon s)\bar{\theta}^* = 0, \tag{28}$$

$$\frac{\partial^2 \bar{\theta}^*}{\partial x^2} - (q^2 + s + \tau_0 s^2)\bar{\theta}^* - \varepsilon s(1 + m\tau_0 s)\frac{\partial \bar{u}^*}{\partial x} + \mathrm{i}q\varepsilon s(1 + m\tau_0 s)\bar{v}^* + (1 + m\tau_0 s)\overline{Q}^* = 0, \tag{29}$$

$$\bar{\sigma}^*_{xx} = \beta_0^2 \frac{\partial \bar{u}^*}{\partial x} + \mathrm{i}q(\beta_0^2 - 2)\bar{v}^* - \beta^2(1 + \upsilon s)\bar{\theta}^*, \tag{30}$$

$$\bar{\sigma}^*_{yy} = (\beta_0^2 - 2)\frac{\partial \bar{u}^*}{\partial x} + \mathrm{i}q\beta_0^2\bar{v}^* - \beta^2(1 + \upsilon s)\bar{\theta}^*, \tag{31}$$

$$\bar{\sigma}^*_{zz} = (\beta_0^2 - 2)\left(\frac{\partial \bar{u}}{\partial x} - \mathrm{i}q\bar{v}^*\right) - \beta^2(1 + \upsilon s)\bar{\theta}^*, \tag{32}$$

$$\bar{\sigma}_{xy} = \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right). \tag{33}$$

Now, we will apply the following Fourier transforms without heat source:

$$\bar{u}^*_c(p, q, s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \bar{u}^*(x, q, s) \cos px\,\mathrm{d}x,$$

$$\bar{v}^*_s(p, q, s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \bar{v}^*(x, q, s) \sin px\,\mathrm{d}x,$$

$$\bar{\theta}^*_s(p, q, s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \bar{\theta}^*(x, q, s) \sin px\,\mathrm{d}x,$$

where

$$\bar{u}^*(x, q, s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \bar{u}^*_c(p, q, s) \cos px\,\mathrm{d}p,$$

$$\bar{v}^*(x, q, s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \bar{v}^*_s(p, q, s) \sin px\,\mathrm{d}p,$$

$$\bar{\theta}^*(x, q, s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \bar{\theta}^*_s(p, q, s) \sin px\,\mathrm{d}p.$$

Hence, we obtain

$$[p^2\beta^2 + \alpha_0 s^2 + q^2]\bar{u}_c^* + \mathrm{i}qp(\beta^2 - 1)\bar{v}_s^* + \beta^2 p(1 + \upsilon s)\bar{\theta}_s^* = \sqrt{\frac{2}{\pi}}[-\beta^2 \bar{u}_0'^* + \mathrm{i}q(\beta^2 - 1)\bar{v}_0^* + \beta^2(1 + \upsilon s)\bar{\theta}_0^*], \quad (34)$$

$$\mathrm{i}qp(\beta^2 - 1)\bar{u}_c^* + [p^2 + \alpha_0 s^2 + q^2\beta^2]\bar{v}_s^* + \mathrm{i}\beta^2 q(1 + \upsilon s)\bar{\theta}_s^* = -\sqrt{\frac{2}{\pi}} p\bar{v}_0^*, \quad (35)$$

$$\varepsilon s(1 + m\tau_0 s)p\bar{u}_c^* + \mathrm{i}q\varepsilon s(1 + m\tau_0 s)\bar{v}_s^* - [p^2 + q^2 + s + \tau_0 s^2]\bar{\theta}_s^* = -\sqrt{\frac{2}{\pi}} p\bar{\theta}_0^*, \quad (36)$$

where

$$\bar{u}_0'^* = \frac{\partial \bar{u}^*(0,q,s)}{\partial x}, \quad \bar{v}_0^* = \bar{v}^*(0,q,s) \quad \text{and} \quad \bar{\theta}_0^* = \bar{\vartheta}^*(0,q,s).$$

Solving the above equations, we get

$$\bar{u}_c^* = \sqrt{\frac{2}{\pi}}\left[\frac{u_1}{p^2 + p_1^2} + \frac{u_2}{p^2 + p_2^2} + \frac{u_3}{p^2 + p_3^2}\right], \quad (37)$$

where

$$u_1 = \frac{l_{11}p_1^4 - l_{12}p_1^2 + l_{13}}{(p_2^2 - p_1^2)(p_3^2 - p_1^2)},$$
$$u_2 = \frac{l_{11}p_2^4 - l_{12}p_2^2 + l_{13}}{(p_1^2 - p_2^2)(p_3^2 - p_2^2)},$$
$$u_3 = \frac{l_{11}p_3^4 - l_{12}p_3^2 + l_{13}}{(p_1^2 - p_3^2)(p_2^2 - p_3^2)},$$
$$l_{11} = -\bar{u}_0'^*,$$
$$l_{12} = -(\alpha_0 s^2 + \beta^2 q^2 + \delta_3 + q^2)\bar{u}_0'^* + \mathrm{i}q\left(\frac{\alpha_0 s^2(\beta^2 - 1)}{\beta^2} + (\beta^2 q^2 - \delta_1\delta_2\varepsilon s - q^2)\right)\bar{v}_0^* + \delta_1(\beta^2 q^2 + \delta_3)\bar{\theta}_0^*, \quad (38)$$
$$l_{13} = -(\alpha_0 s^2(\delta_3 + q) + \beta^2 q^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2))\bar{u}_0'^* + \frac{\mathrm{i}q(\beta^2 - 1)}{\beta^2}(\alpha_0 s^2(\delta_3 + q) + \beta^2 q^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2))\bar{v}_0^* + \delta_1(\alpha_0 s^2(\delta_3 + q) + \beta^2 q^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2))\bar{\theta}_0^*,$$
$$\bar{v}_s^* = \sqrt{\frac{2}{\pi}}\left[\frac{v_1 p}{p^2 + p_1^2} + \frac{v_2 p}{p^2 + p_{21}^2} + \frac{v_3 p}{p^2 + p_3^2}\right],$$

where

$$v_1 = \frac{l_{21}p_1^4 - l_{22}p_1^2 + l_{23}}{(p_2^2 - p_1^2)(p_3^2 - p_1^2)},$$
$$v_2 = \frac{l_{21}p_2^4 - l_{22}p_2^2 + l_{23}}{(p_1^2 - p_2^2)(p_3^2 - p_2^2)},$$
$$v_3 = \frac{l_{21}p_3^4 - l_{22}p_3^2 + l_{23}}{(p_1^2 - p_3^2)(p_2^2 - p_3^2)},$$

$$l_{21} = \bar{v}_0^*,$$

$$l_{22} = \mathrm{i}q(1-\beta^2)\bar{u}_0'^* + \left(\frac{\alpha_0 s^2}{\beta^2} - (\beta^2 q^2 - \delta_1\delta_2\varepsilon s - \delta_3 - 3q^2)\right)\bar{v}_0^* + \mathrm{i}q\delta_1\bar{\theta}_0^*,$$

$$l_{23} = -\mathrm{i}q(\beta^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2) - \delta_3 - q^2)\bar{u}_0'^* + \left(\frac{\alpha_0 s^2(\delta_3+q^2)}{\beta^2} - q^2(\beta^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2) - \delta_1\delta_2\varepsilon s - 2(\delta_3+q^2))\right)\bar{v}_0^* + (i\alpha_0 s^2 q\delta_1 + \mathrm{i}q\delta_1(\beta^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2) - \delta_3))\bar{\theta}_0^*,$$

and

$$\bar{\theta}_s^* = \sqrt{\frac{2}{\pi}}\left[\frac{\theta_1 p}{p^2+p_1^2} + \frac{\theta_2 p}{p^2+p_2^2} + \frac{\theta_3 p}{p^2+p_3^2}\right], \tag{39}$$

where

$$\theta_1 = \frac{l_{31}p_1^4 - l_{32}p_1^2 + l_{33}}{(p_2^2-p_1^2)(p_3^2-p_1^2)},$$

$$\theta_2 = \frac{l_{31}p_2^4 - l_{32}p_2^2 + l_{33}}{(p_1^2-p_2^2)(p_3^2-p_2^2)},$$

$$\theta_3 = \frac{l_{31}p_3^4 - l_{32}p_3^2 + l_{33}}{(p_1^2-p_3^2)(p_2^2-p_3^2)},$$

$$l_{31} = \bar{\theta}_0^*,$$

$$l_{32} = -\varepsilon s\delta_2\bar{u}_0'^* + i\varepsilon s\delta_2 q\bar{v}_0^* + \left(\frac{\alpha_0 s^2(\beta^2+1)}{\beta^2} + (\delta_1\delta_2\varepsilon s + 2q^2)\right)\bar{\theta}_0^*,$$

$$l_{33} = -\delta_2\varepsilon s(\alpha_0 s^2 + q^2)\bar{u}_0'^* + \mathrm{i}qs\varepsilon\delta_2(\alpha_0 s^2 + q^2)\bar{v}_0^* + \frac{1}{\beta^2}(\alpha_0^2 s^4 + \alpha_0 s^2(\beta^2(\delta_1\delta_2\varepsilon s + q^2) + q^2))\bar{\theta}_0^*.$$

We have already used the following parameters:

$$p_1^2 = \alpha_0 s^2 + q^2,$$

$$p_2^2 = \frac{1}{2}[L + \sqrt{L^2 - 4LM}],$$

$$p_3^2 = \frac{1}{2}[L - \sqrt{L^2 - 4LM}],$$

$$L = \left[\frac{\alpha_0 s^2}{\beta^2} + (\delta_1\delta_2\varepsilon s + \delta_3 + 2q^2)\right],$$

$$M = \left[\frac{\alpha_0 s^2(\delta_3+q^2)}{\beta^2} + q^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2)\right],$$

$$\delta_1 = 1 + \upsilon s, \quad \delta_2 = 1 + \tau_0 m s \quad \text{and} \quad \delta_3 = s + \tau_0 s^2.$$

Now, by using the following integrals:

$$\int_0^\infty \frac{\cos px}{p^2+k^2}\,\mathrm{d}p = \frac{\pi}{2}\frac{\mathrm{e}^{-kx}}{k} \quad \text{and} \quad \int_0^\infty \frac{p\sin px}{p^2+k^2}\,\mathrm{d}p = \frac{\pi}{2}\mathrm{e}^{-kx},$$

we get

$$\bar{u}^*(x,q,s) = \frac{u_1}{p_1}\mathrm{e}^{-p_1x} + \frac{u_2}{p_2}\mathrm{e}^{-p_2x} + \frac{u_3}{p_3}\mathrm{e}^{-p_3x}, \tag{40}$$

$$\bar{v}^*(x,q,s) = v_1\mathrm{e}^{-p_1x} + v_2\mathrm{e}^{-p_2x} + v_3\mathrm{e}^{-p_3x}, \tag{41}$$

$$\bar{\theta}^*(x,q,s) = \theta_1\mathrm{e}^{-p_1x} + \theta_2\mathrm{e}^{-p_2x} + \theta_3\mathrm{e}^{-p_3x}. \tag{42}$$

We can get the constitutive equations as following:

$$\bar{\sigma}^*_{xx}(x,q,s) = -\sum_{j=1}^{3}[\beta_0^2u_j + \mathrm{i}q(\beta_0^2-2)v_j + \beta^2\delta_1\theta_j]\mathrm{e}^{-p_jx}, \tag{43}$$

$$\bar{\sigma}^*_{yy}(x,q,s) = -\sum_{j=1}^{3}[(\beta_0^2-2)u_j - \mathrm{i}q\beta_0^2v_j + \beta^2\delta_1\theta_j]\mathrm{e}^{-p_jx}, \tag{44}$$

$$\bar{\sigma}^*_{zz}(x,q,s) = -\sum_{j=1}^{3}[(\beta_0^2-2)u_j + \mathrm{i}q(\beta_0^2-2)v_j + \beta^2\delta_1\theta_j]\mathrm{e}^{-p_jx}, \tag{45}$$

$$\bar{\sigma}^*_{xy}(x,q,s) = -\sum_{i=1}^{3}\left[\frac{\mathrm{i}q}{p_j}u_j + p_jv_j\right]\mathrm{e}^{-p_jx}. \tag{46}$$

5. Application

We consider the problem of a half-space, which is defined in the region Ω defined as following:

$$\Omega = \{(x,y,z):\ 0 \leqslant x < \infty, -\infty < y < \infty, -\infty < z < \infty\}.$$

We consider a thermal chock in the side of the region at $x = 0$, then

$$\theta(0,y,t) = H(t)F(y).$$

After applying Laplace and Fourier transforms as we defined before, we obtain

$$\bar{\theta}^*_0(0,q,s) = \frac{F^*}{s}. \tag{47}$$

We consider the body satisfies the following mechanical conditions:

$$v(0,y,t) = 0, \quad \text{and} \quad u'(0,y,t) = 0.$$

After applying Laplace and Fourier transforms, we obtain

$$\bar{v}^*_0 = \bar{v}^*(0,q,s) = 0, \tag{48}$$

$$\bar{u}'^*_0 = \frac{\partial\bar{u}^*(0,q,s)}{\partial x} = 0. \tag{49}$$

That completes the solution of the Eqs. (40)–(42) in the transformed domain as following:

$$\bar{u}^*(x,q,s) = \frac{u_{10}}{p_1}\mathrm{e}^{-p_1x} + \frac{u_{20}}{p_2}\mathrm{e}^{-p_2x} + \frac{u_{30}}{p_3}\mathrm{e}^{-p_3x}, \tag{50}$$

where

$$
\begin{aligned}
u_{10} &= \frac{l_{110}p_1^4 - l_{120}p_1^2 + l_{130}}{(p_2^2 - p_1^2)(p_3^2 - p_1^2)}, \\
u_{20} &= \frac{l_{110}p_2^4 - l_{120}p_2^2 + l_{130}}{(p_1^2 - p_2^2)(p_3^2 - p_2^2)}, \\
u_{30} &= \frac{l_{110}p_3^4 - l_{120}p_3^2 + l_{130}}{(p_1^2 - p_3^2)(p_2^2 - p_3^2)}, \\
l_{110} &= 0, \\
l_{120} &= \delta_1(\beta^2 q^2 + \delta_3)\frac{F^*}{s}, \\
l_{130} &= \delta_1(\alpha_0 s^2(\delta_3 + q) + \beta^2 q^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2))\frac{F^*}{s}, \\
\bar{v}^*(x,q,s) &= v_{10}e^{-p_1 x} + v_{20}e^{-p_2 x} + v_{30}e^{-p_3 x},
\end{aligned} \tag{51}
$$

where

$$
\begin{aligned}
v_{10} &= \frac{l_{210}p_1^4 - l_{220}p_1^2 + l_{230}}{(p_2^2 - p_1^2)(p_3^2 - p_1^2)}, \\
v_{20} &= \frac{l_{210}p_2^4 - l_{220}p_2^2 + l_{230}}{(p_1^2 - p_2^2)(p_3^2 - p_2^2)}, \\
v_{30} &= \frac{l_{210}p_3^4 - l_{220}p_3^2 + l_{230}}{(p_1^2 - p_3^2)(p_2^2 - p_3^2)}, \\
l_{210} &= 0, \\
l_{220} &= \mathrm{i}q\delta_1\frac{F^*}{s}, \\
l_{230} &= (\mathrm{i}\alpha_0 s^2 q\delta_1 + \mathrm{i}q\delta_1(\beta^2(\delta_1\delta_2\varepsilon s + \delta_3 + q^2) - \delta_3))\frac{F^*}{s}, \\
\bar{\theta}^*(x,q,s) &= \theta_{10}e^{-p_1 x} + \theta_{20}e^{-p_2 x} + \theta_{30}e^{-p_3 x},
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
\theta_{10} &= \frac{l_{310}p_1^4 - l_{320}p_1^2 + l_{330}}{(p_2^2 - p_1^2)(p_3^2 - p_1^2)}, \\
\theta_{20} &= \frac{l_{310}p_2^4 - l_{320}p_2^2 + l_{330}}{(p_1^2 - p_2^2)(p_3^2 - p_2^2)}, \\
\theta_{30} &= \frac{l_{310}p_3^4 - l_{320}p_3^2 + l_{330}}{(p_1^2 - p_3^2)(p_2^2 - p_3^2)}, \\
l_{310} &= \frac{F^*}{s}, \\
l_{320} &= \left(\frac{\alpha_0 s^2(\beta^2 + 1)}{\beta^2} + (\delta_1\delta_2\varepsilon s + 2q^2)\right)\frac{F^*}{s}, \\
l_{330} &= (\alpha_0^2 s^4 + \alpha_0 s^2(\beta^2(\delta_1\delta_2\varepsilon s + q^2) + q^2))\frac{F^*}{\beta^2 s}.
\end{aligned}
$$

The constitutive equations take the forms

$$\bar{\sigma}^*_{xx}(x,q,s) = -\sum_{j=1}^{3}[\beta_0^2 u_{j0} + \mathrm{i}q(\beta_0^2-2)v_{j0} + \beta^2\delta_1\theta_{j0}]\mathrm{e}^{-p_j x}, \tag{53}$$

$$\bar{\sigma}^*_{yy}(x,q,s) = -\sum_{j=1}^{3}[(\beta_0^2-2)u_{j0} - \mathrm{i}q\beta_0^2 v_{j0} + \beta^2\delta_1\theta_{j0}]\mathrm{e}^{-p_j}x, \tag{54}$$

$$\bar{\sigma}^*_{zz}(x,q,s) = -\sum_{i=1}^{3}[(\beta_0^2-2)u_{j0} + \mathrm{i}q(\beta_0^2-2)v_{j0} + \beta^2\delta_1\theta_{j0}]\mathrm{e}^{-p_j x}, \tag{55}$$

$$\bar{\sigma}^*_{xy}(x,q,s) = -\sum_{j=1}^{3}\left[\frac{\mathrm{i}q}{p_j}u_{j0} + p_j v_{j0}\right]\mathrm{e}^{-p_j x}. \tag{56}$$

6. Inversion of the Laplace transform

To obtain the solution of the problem in the physical domain (x, y, t), we have to invert the iterated transforms in Eqs. (50)–(56).

These expressions can be formally expressed as functions of x and the parameter of the Fourier and Laplace transforms q and s of the form $\bar{f}^*(x,q,s)$ (see e.g. Honig and Hirdes, 1984).

First, we invert the Fourier transform using the inversion formula given previously. This gives the Laplace transform expression $\bar{f}(x,y,s)$ of the function $f(x,y,t)$ as

$$\bar{f}(x,y,s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}qy}f(x,q,s)\,\mathrm{d}q$$
$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty}(\cos(qy)f_{\mathrm{e}} + \mathrm{i}\sin(qy)f_{\mathrm{o}})\,\mathrm{d}q,$$

where f_{e} and f_{o} denote the even and the odd parts of the function $\bar{f}^*(x,q,s)$ respectively.

We shall now outline the numerical inversion method used to find the solution in the physical domain. For fixed values of x, y, and q the function inside braces in the last integral can be considered as a Laplace transform $\bar{g}(s)$ of some function $g(t)$.

The inversion formula for the Laplace transform can be written as

$$g(t) = \frac{1}{2\pi\mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}\mathrm{e}^{st}\bar{g}(s)\,\mathrm{d}s,$$

where c is an arbitrary real number greater than all the parts of the singularities $\bar{g}(s)$. Taking $s = c + \mathrm{i}y$, the above integral takes the form

$$g(t) = \frac{\mathrm{e}^{ct}}{2\pi}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}ty}\bar{g}(c+\mathrm{i}y)\,\mathrm{d}y.$$

Expanding the function $h(t) = \exp(-ct)g(t)$ in a Fourier series in the interval $[0, 2L]$, we obtain the approximate formula.

$$g(t) = g_\infty(t) + E_D,$$

where

$$g_\infty(t) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty}c_k \quad \text{for } 0 \leqslant t \leqslant 2L, \tag{57}$$

and

$$c_k = \frac{e^{ct}}{L} Re[e^{ik\pi t/L} g(c + ik\pi t/L)]. \tag{58}$$

E_D, the discretization error, can be made arbitrary small by choosing c large enough.

Since the infinite series in Eq. (57) can be summed up to finite number N of terms, the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{1}{2} c_0 + \sum_{k=1}^{N} c_k \quad \text{for } 0 \leqslant t \leqslant 2L. \tag{59}$$

Using the above formula to evaluate $g(t)$, we introduce a truncation error E_T which, must be added to the discretization error to produce the total approximation error.

Two methods are used to reduce the total error. First the "Korrecktur" method is used to reduce the discretization error. Next, the ε algorithm is used to reduce the truncation error and hence to accelerate convergence.

The Korrecktur method uses the following formula to evaluate the function $g(t)$:

$$g(t) = g_\infty(t) - e^{2cL} g_\infty(2L + t) + E'_D,$$

where the discretization error $|E'_D| \ll |E_D|$. Thus, the approximate value of $g(t)$ becomes

$$g_{NK}(t) = g_N(t) - e^{-2cL} g_{N'}(2L + t). \tag{60}$$

N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in Eq. (59). Let $N = 2q + 1$ where q is a natural number, and let

$$s_m = \sum_{k=1}^{m} c_k$$

be the sequence of partial sums of Eq. (59). We define the ε-sequence by

$$\varepsilon_{o,m} = 0 \quad \varepsilon_{1,m} = 0,$$

and

$$\varepsilon_{p+1,m} = \frac{\varepsilon_{p-1,m+1} + 1}{\varepsilon_{p,m+1} - \varepsilon_{p,m}}, \quad p = 1, 2, 3, \ldots$$

it can be shown that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \varepsilon_{5,1}, \ldots, \varepsilon_{N,1}$, converges to $f(x, y, t) + E_D - c_0/2$ faster than the sequence of partial sums s_m $(m = 1, 2, 3, \ldots)$.

The actual procedure used to invert the Laplace transform consists of using Eq. (60) together with the ε-algorithm. The values of c and L are chosen according to the criteria outlined in Honig and Hirdes (1984).

7. Numerical results

The function $F(y)$ representing the thermal shock was taken as $F(y) = H(a - |y|)$ which gives

$$F^*(q) = \sqrt{\frac{2}{\pi}} \frac{\sin(qa)}{q},$$

where H denotes Heaviside step function.

The copper material was taken for chosen purpose of numerical evaluations. The constant of the problem were taken as in Ezzat (2004):

$$k = 386\ \text{N/K s},\quad \alpha_T = 1.78 \times 10^{-5}\ \text{K}^{-1},\quad C_E = 383.1\ \text{m}^2/\text{K s}^2,\quad \eta_0 = 8886.73\ \text{m/s}^2,$$
$$\mu = 3.86 \times 10^{10}\ \text{N/m}^2,\quad \lambda = 7.76 \times 10^{10}\ \text{N/m}^2,\quad c_1 = 1.39 \times 10^{-5},\quad \rho = 8954\ \text{kg/m}^3,$$
$$\varepsilon_0 = (10)^{-9}/(36\pi)\ \text{C}^2/\text{N m}^2,\quad \mu_0 = 4\pi \times 10^{-7}\ \text{N m s}^2/\text{C}^2,\quad H_0 = 1\ \text{C/m s},\quad \tau_0 = 0.02,$$
$$\upsilon = 0.03,\quad T_0 = 293\ \text{K},\quad \beta_0^2 = 2.01,\quad \beta^2 = 3.5,\quad \varepsilon = 0.0168,\quad a = 1,$$

and the computations were performed for one value of time, namely for $t = 0.01$. These computations were carried out in the coupled theory ($\tau_0 = v = 0$), in Lord–Shulman theory ($m = 1,\ v = 0,\ \tau_0 = 0.02$) and in Green–Lindsay theory ($\tau_0 = m = 0,\ v = 0.03$), when the medium is a perfect electric conductor. The numerical values of the temperature, displacement components and stress components are obtained and represented graphically for these theories. The results are shown in Figs. 1–6. The graph shows the three curves predicted by the three different theories of thermoelasticity.

The phenomenon of finite speeds of propagation is manifested in all these figures. This is expected, since the thermal wave travels with a finite velocity. It should be mentioned in Fig. 1 that the effects of the heating by a thermal shock on $x = 0$ of the half space remain in a bounded region of space in the two generalized theories and does not reach infinity instantaneously. This is not the case when using the equations of coupled thermoelasticity theory where the response to thermal disturbance reaches infinity instantaneously.

The presence of the magnetic field which acts to the perfect conducting elastic medium raises the velocity of the dilatational elastic waves from β_0 to $c_0 = (\beta_0^2 + \alpha_0^2)^{1/2}$, the modified electromagnetic elastic wave is

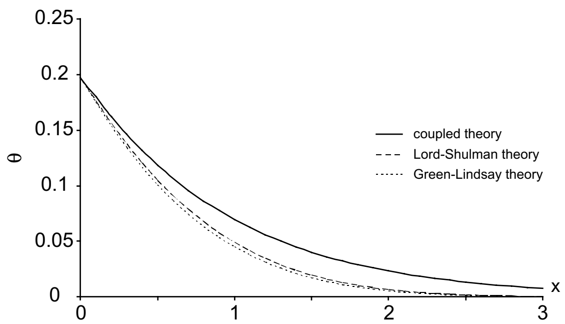

Fig. 1. The temperature distribution.

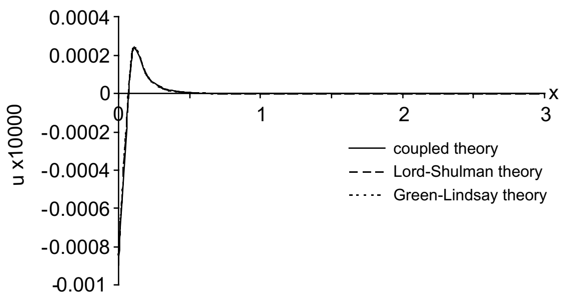

Fig. 2. The horizontal displacement distribution.

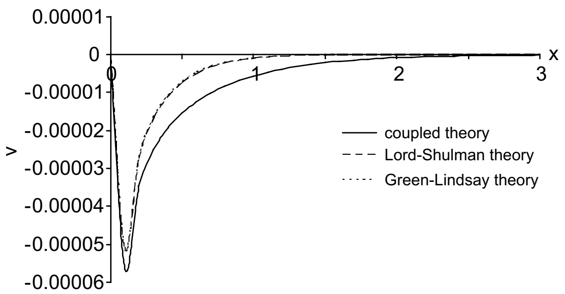

Fig. 3. The vertical displacement distribution.

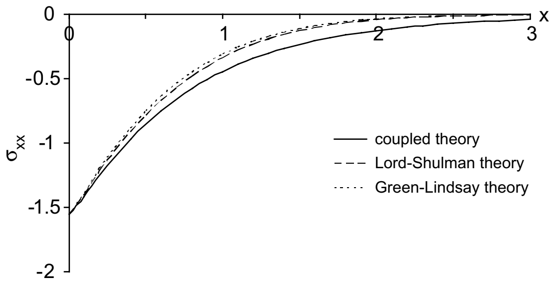

Fig. 4. The stress component distribution.

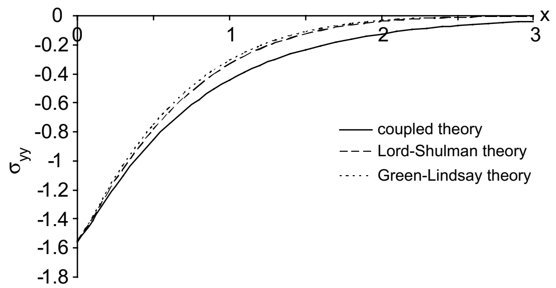

Fig. 5. The stress component distribution.

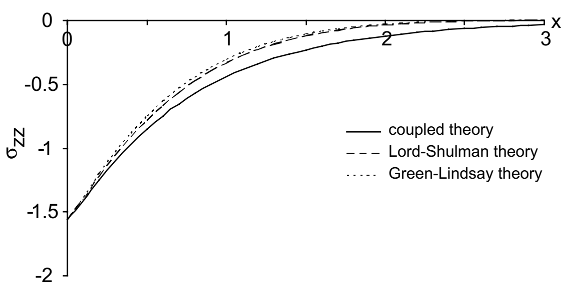

Fig. 6. The stress component distribution.

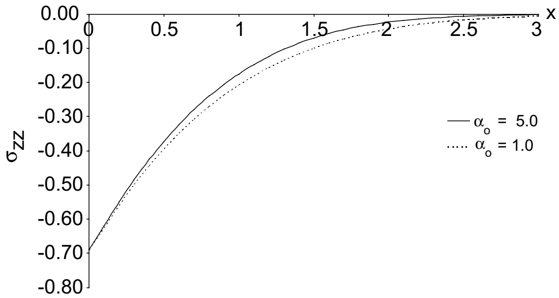

Fig. 7. The effect of the magnetic field on the stress distribution.

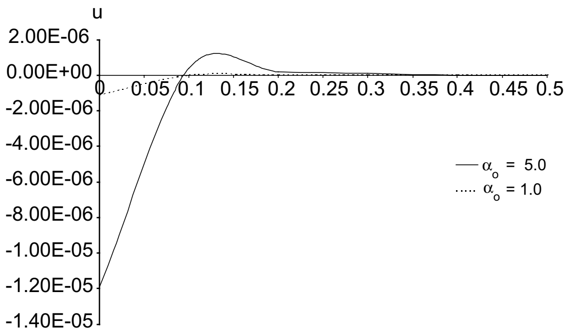

Fig. 8. The effect of the magnetic field on the displacement distribution.

propagated with velocity c_0, and that is, with the same velocity as the modified elastic wave that produces a jump in stress (Figs. 7 and 8).

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